Assignment 6

Coverage: 15.8 in Text.

Exercises: 15.8. No 1, 3, 7, 9, 12, 14, 15, 16, 19, 20, 27.

Submit no. 7, 9, 12, 16, 19 by October 25.

Supplementary Problems

1. Find the volume of the ball $x^2 + y^2 + z^2 + w^2 \le R^2$ in $\mathbb{R}^4$ by the formula
$$\text{vol} = \int_{-R}^{R} |B_w|\, dw \ ,$$
where $|B_w|$ is the volume of the cross section of the ball at height w. The answer is $\pi^2 R^4/2$.

2. Let D be a region in the plane which is symmetric with respect to the origin, that is, $(x,y) \in D$ if and only if $(-x,-y) \in D$. Show that
$$\iint_D f(x,y)\, dA(x,y) = 0 \ ,$$
when f is odd, that is, $f(-x,-y) = -f(x,y)$ in D. This problem appears in Ex 4. Now you are asked to apply the change of variables formula in two dimension.

3. The rotation by an angle θ in anticlockwise direction is given by $(x,y) = (\cos\theta\ u - \sin\theta\ v, \sin\theta\ u + \cos\theta\ v)$. Verify that rotation leaves the area unchanged.

4. Consider the map $(u,v) \mapsto (x,y) = (u^2, v)$ which maps the square $R_1 = [-1,1] \times [0,1]$ onto $R_2 = [0,1] \times [0,1]$. Show that
$$\iint_{R_2} f(x,y) dA(x,y) \ne \iint_{R_1} f(u^2,v) \left| \frac{\partial(x,y)}{\partial(u,v)} \right| dA(u,v) \ .$$
(Hint: It suffices to take $f(x,y) \equiv 1$.) Why?